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Following the steps of Spanish Mathematical
Analysis: From Cauchy to Weierstrass between 1880 and 1914

## 3 Clariana and his lecture notes on Mathematical Analysis: Complex Analysis

Lauro Clariana Ricart (1842-1916) spent his life in Catalonia, mostly in Barcelona and Tarragona, the capital city of the neighbouring province of the same name. He studied in Barcelona the courses needed to become Licenciado (1872) in the so called Ciencias Exactas, the name given in Spain to Mathematics for more than a century. One year later he obtained the Doctorate in the exact Sciences at the same university. Clariana was, for a long period between 1865 and 1916, a prolific writer on many topics ranging from Mathematical Analysis and Rational Mechanics to Music and the construction of several machines. A detailed biographical account and a complete list of Clariana's writings –some of them unpublished- can be found in (Clariana-Clarós, 1993).

Clariana taught Integral and Differential Calculus, and Rational Mechanics and was one of the few Spaniards who participated in several International Congresses and Meetings. He travelled to Paris, Brussels, München, and Freiburg between 1888 and 1900, and in 1888 he was awarded in Paris a prize for his Memoir "*On the spirit of Mathematics in the modern times*". Clariana was the author of two books on Mathematical Analysis for the use of students at the Escuela Superior de Ingeniería Industrial in Barcelona. They were published in 1892 and 1893 under the titles *Resumen de las lecciones de Cálculo Diferencial e Integral* (Clariana Ricart, 1892b) (Resumen from now on), and *Complemento a los elementos de los Cálculos* (Clariana Ricart, 1892a) (Complemento in what follows). They appeared as lithographed handwritten lecture notes, and a joint edition under the title *Conceptos fundamentals de Análisis Matemático* appeared in a more normal printing style the year 1903 (Clariana Ricart, 1903). A small favourable review of this last book appeared in the Bulletin of the American Mathematical Society the year 1904 (McFarlane, 1904).

Resumen is a general introduction to Analysis, and at the very beginning, in page 5, Clariana makes his position clear[1]:

*Because the infinitely small and the infinitely large are the only elements that can become the basis of quantity in Mathematics, synthesised in the finite ones, we shall admit three categories of quantity under the following forms:*
*(1) That of the infinitely small.*
*(2) That of the finite.*
*(3) That of the infinitely large.*
*And these are the only true concepts of quantity that are directly connected to the Leibnizian idea of a differential*

[1] Siendo los indefinidamente pequeños, así como los indefinidamente grandes, los únicos elementos que pueden constituir la base de la cantidad en matemáticas, sintetizados en lo finito, admitiremos en esta ciencia tres categorías de cantidad, bajo la forma siguiente:
(1) Correspondiente a los indefinidamente pequeños.
(2) Correspondiente a lo finito.
(3) Correspondiente a los indefinidamente grandes.
Estos son los únicos y verdaderos conceptos de cantidad que se enlazan directamente con la diferencial de Leibniz

A very long introduction (Prolegómenos) of 45 pages is offered on the various classes of numbers and functions, as well as on the foundations and the history of the infinitesimal method, where the author summons Descartes, Johann Bernouilli and Cournot, and indeed Newton, Leibniz, and D'Alembert. The rest of the book is a classical treatise on the usual topics on Differential and Integral Calculus presented in a straightforward way. Theorems are not highlighted and proofs are not distinctly offered. The emphasis is on the succession of useful and applicable formulas, with a few examples spread over the text. This makes the book very readable and surprisingly modern even to today's standards. Complemento has a different flavour: It is a compilation of loosely knit topics in higher Analysis. Clariana declares in a brief introduction:

*The aim of this book is to present what we should call 'modern theories of the infinitesimal and integral calculus', not because some of them are recent ones, but because they have not yet been presented in the Spanish education.*

The first two chapters are devoted to "Infinitesimally Small Triangles" and "Orders of Comparison for Curves", where the fundamental Leibnizian triangle is explained and applied in depth, as well as the idea of the order of an infinitesimal is deeply applied to the study of different elements of curves.

With this equipment, the book follows with a study of Classical Differential Geometry. On the remaining chapters, a variety of topics is included. There are the Euler-McLaurin summation formula, special functions and elliptic integrals... To summarise; it is a simplified version of the usual second volume in the classical French treatises that clearly inspired the author, and the style is the same of Resumen. In basic questions, Clariana holds the same opinions of Archilla. As an example, the definition of a continuous function on an interval reads:

*The function y = F(x) is continuous between the values a and b attributed to x if its values pass from one value to another through values that differ between them as little as desired.*

But the main feature in Clariana's work is that he is the first author to introduce in print Complex Analysis in Spain. He plainly states that "a complex quantity has the form x + yi , where x and y are real quantities" and goes on by explaining that Gauss was the first to speak of yi as imaginary and that Cauchy denoted as imaginary the whole complex quantity. Of course he points out that when x and y are variable quantities, then x+yi is a complex variable and that a complex quantity is infinitesimally small (large) if the real part x and the imaginary part y are infinitesimally small (large) quantities. Continuity of a complex quantity is, of course, assessed form the continuity of its component real variables. Then, a standard theory follows.

It must be noted that before Clariana no Spanish mathematician had studied complex quantities as the object of Analysis. Only algebraic, geometric or arithmetic considerations had been made in Spain on these numbers, and for nearly forty years the source book was the rather obscure Teoría transcendental de las cantidades imaginarias, the posthumously edited work (1865) of José María Rey Heredia (1818-1861) who inspired several developments, especially in the presentation of Analytic Geometry. The work of Rey Heredia and some of his followers has been extensively studied elsewhere by the authors (Pacheco Castelao et al., 2006a).

## 6 Conclusions and views

In this paper the authors show that the work of several mathematicians and engineers must be acknowledged in the enterprise of introducing rigour in the teaching and spreading of Mathematical Analysis in Spanish universities and engineering schools. The most representative four personalities and books have been dealt with, highlighting their achievements:
(1) Archilla is the first Spanish mathematician to introduce the Cauchy style in his book Principios de Cálculo Diferencial.

(2) Clariana was the first to present complex analysis, as well as the definition of continuity of a complex function.
(3) Villafañe made a most interesting effort when he presented

- The total derivative of a complex function.
- Several definitions related with complex functions, among them that of a holomorphic function.
- The relationship between the existence of a derivative and the Cauchy-Riemann conditions for continuous functions.
- Absolute values as a tool in proofs and definitions.

(4) Pérez de Muñoz presented for the first time in Spanish a construction of the real field and the proof of the continuity of a derivable function.

References (resumen)

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